MEASUREMENT OF VELOCITY VECTOR IN GAS STREAM

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A method of determining the three-dimensional vector of mean velocity in a gas stream with a three-wire probe and a constant-current thermoanemometer has been developed and experimentally evaluated.

In many applications in aerodynamics it is now necessary to determine experimentally the field distribution of the mean-velocity vector in a moving medium. In this study the problem of measuring the mean-velocity vector in a gas stream with an inaccuracy not exceeding 10% has been solved during development of a new chemical technology. The turbulence level in a stream did not exceed 5% of the mean velocity. The thermoanemometric method was used, this low level of turbulence and no need for measurement of fluctuations having determined the selection of a simple constant-current instrument which is easy to construct and adjust. There already have been proposed algorithms for calculating the mean-velocity vector from measurements with a constant-temperature thermoanemometer [1]. Use of a constant-current thermoanemometer features several peculiarities.

The instrument transducer (Fig. 1) consists of three hot wires 1, 2, 3 disposed orthogonally to one another along the axes of the local Cartesian system of coordinates. Over other instrument transducers serving the same purpose (rotating one and cruciform one) such a three-wire instrument offers the advantage of yielding data in real time and easily processable, all components of the mean-velocity vector and its modulus being determined simultaneously.

The wires were made as nearly identical as possible, of gold-plated tungsten d = 8 μm in diameter and l = 4 mm long with the following characteristics: electrical resistance (at t = 20°C) R_a = 3.4-3.6 Ω, maximum overheat current 75 mA, and range of gas velocity measurement 0.2-60 m/sec.

The wires were welded to needle-shaped holders made of metal and serving as current leads. The axis of the transducer housing was made to be the axis of symmetry of the transducer assembly. The small linear dimensions of the wires have made it feasible to attain a sufficiently high space resolution and, at the same time, a relatively excellent manufacturability.

Let us estimate the error in determining the components of the mean-velocity vector due to inaccurate orthogonality of the transducer wires. For simplicity, we will consider two-dimensional flow (Fig. 2). In this case the components of the velocity vector are

$$v_x = v\cos\theta,\ v_y = v\sin\theta,$$

where θ is the angle between vector v and the OX axis.

Let the OY axis and the OX axis form an angle $(90+\varphi)^\circ$, as shown in Fig. 2. Then the v_y' component of velocity is

$$v_y' = v\sin(\theta - \varphi)\ ,$$

the absolute error is

$$|\Delta v_y| = v\cos(\theta - \varphi)\,\Delta\varphi,$$

and the relative error is

$$\frac{|\Delta v_y|}{v_y} = \mathrm{ctg}\,(\theta - \varphi)\,\Delta\varphi. \quad (1)$$

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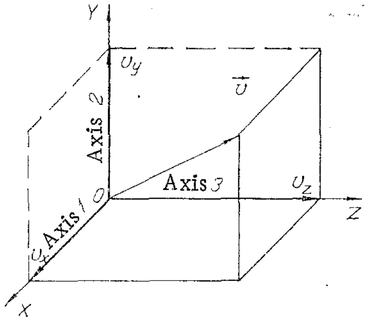

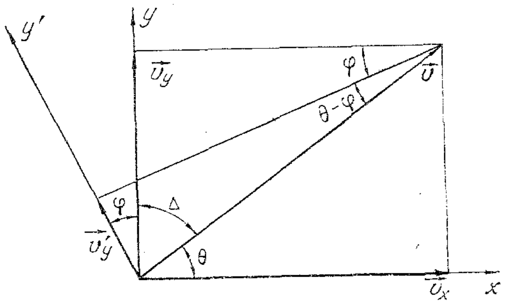

Fig. 1 Fig. 2

Fig. 1. Three-wire probe and corresponding local system of coordinates.

Fig. 2. Determining the orthogonality requirements for the probe wires.

At the present time probes are series manufactured with a nonorthogonality of wires $\Delta\varphi = 2\text{-}3°$. For the worst case of $\Delta\varphi = 3°$ we will determine the maximum angle θ between the direction of flow and the probe wire along the OX axis at which the relative error will be smaller than a stipulated 10%. We can do this by solving Eq. (1) for θ

$$0.1 = \mathrm{ctg}\,(\theta - 3°)\,0.052,$$

which yields θ = 31° ($\Delta\varphi$ is here expressed in radians for calculating θ).

With a given nonorthogonality $\Delta\varphi = 3°$, therefore, the stipulated 10% accuracy in determining the mean-velocity vector is attained at an angle of at least 31-32° between the velocity vector and the wire designated as lying on the OX axis. Consequently, the operating range of the probe will lie within some solid angle limited by the nonorthogonality. This solid angle has for each wire an aperture of $0 < \Delta < 55\text{-}60°$ at the vertex, and in the case of three-wire probe the operating range will be further narrowed down to the ranges common for all three wires.

The operating range of the probe is analogously limited by the dependence of the effective velocity v_{eff} of the stream which cools a wire on the angle ψ between the direction of flow and the plane normal to the axis of that wire.

As has been demonstrated [2], the form of this dependence is determined by the parameter l/d, in our case $l/d = 500$ and for such a value of this parameter $v_{eff} = f(\psi)$ becomes

$$v_{eff} = v\cos\psi. \tag{2}$$

This relation ensures sufficient accuracy for one wire within the $0 < \psi < 60°$ range, namely when angles Δ and ψ are equal at a stipulated 10% accuracy and a given 3° nonorthogonality. Accordingly, the limits within which relation (2) holds true will not narrow down the operating range of a three-wire probe defined as a conical sector of space with a vertex angle of 50°.

Each wire in the probe of a constant-current thermoanemometer is connected into one arm of a Wheatstone bridge. Prior to immersion in a stream the thermoanemometer is in dynamic equilibrium and its output voltage corresponds to zero stream velocity. Immersion of the thermoanemometer in a stream causes an unbalance in the bridge circuit.

The well-known King equation [3] for this thermoanemometer operation in the constant-current mode is

$$\frac{R_w}{R_w - R_g} = A + Bv^n, \tag{3}$$

with n = 0.5 selected in this particular study. The bridge unbalance voltage is amplified in the electronic part of the instrument so that the output voltage E_{out} will be proportional to the change of electrical resistance of the probe

$$E = a\Delta R_w. \tag{4}$$

The quantities R_a, R_w, and a are constant for a given wire at a given overheat temperature and are determined during calibration of the instrument together with the probe. Upon immersion of the instrument transducer in a gas stream, the electrical resistance of the probe changes from R_w at the initial overheat temperature by an amount ΔR_w so that expression (3) can be rewritten as

$$\frac{R_w - \Delta R_w}{R_w - \Delta R_w - R_g} = A + Bv^{0.5}.$$

With the aid of relation (4), this latter expression can be easily reduced to

$$\frac{1}{C - E} = A_1 + B_1 r^{0.5}, \tag{5}$$

where

$$C = a(R_w - R_g); \quad A_1 = \frac{A - 1}{R_g a}; \quad B_1 = \frac{B}{R_g a}. \tag{5a}$$

We have thus obtained the convenient expression (5) relating the output voltage of the thermoanemometer to the velocity of the gas stream. Expressing the velocity v_{eff} for each wire through its components [1], with relation (5) taken into account, we arrive at the system of equations

$$\begin{aligned}
\left(\frac{\frac{1}{C - E_1} - A_1}{B_1}\right)^4 &= k_1^2 v_x^2 + v_y^2 + v_z^2, \\
\left(\frac{\frac{1}{C - E_2} - A_1}{B_1}\right)^4 &= v_x^2 + k_2^2 v_y^2 + v_z^2, \\
\left(\frac{\frac{1}{C - E_3} - A_1}{B_1}\right)^4 &= v_x^2 + v_y^2 + k_3^2 v_z^2
\end{aligned} \tag{6}$$

relating the magnitudes of probe signals E_1, E_2, E_3 to the components of the velocity vector. A solution of this system of equations (6) for v_x, v_y, v_z yields

$$\begin{aligned}
v_x &= \left\{\frac{1}{2 - (k_1^2 + k_2^2 + k_3^2)}\left[\left(\frac{\frac{1}{C - E_2} - A_1}{B_1}\right)^4 (1 - k_3^2) + \left(\frac{\frac{1}{C - E_3} - A_1}{B_1}\right)^4 (1 - k_2^2) - \left(\frac{\frac{1}{C - E_1} - A_1}{B_1}\right)^4\right]\right\}^{1/2}, \\
v_y &= \left\{\frac{1}{2 - (k_1^2 + k_2^2 + k_3^2)}\left[\left(\frac{\frac{1}{C - E_1} - A_1}{B_1}\right)^4 (1 - k_3^2) + \left(\frac{\frac{1}{C - E_3} - A_1}{B_1}\right)^4 (1 - k_1^2) - \left(\frac{\frac{1}{C - E_2} - A_1}{B_1}\right)^4\right]\right\}^{1/2}, \\
v_z &= \left\{\frac{1}{2 - (k_1^2 + k_2^2 + k_3^2)} \cdot \left(\frac{\frac{1}{C - E_2} - A_1}{B_1}\right)^4 (1 - k_1^2) + \left(\frac{\frac{1}{C - E_1} - A_1}{B_1}\right)^4 (1 - k_2^2) - \left(\frac{\frac{1}{C - E_3} - A_1}{B_1}\right)^4\right]\right\}^{1/2},
\end{aligned} \tag{7}$$

and the modulus of the mean-velocity vector

$$\begin{aligned}
v = \Bigg\{\frac{1}{2 - (k_1^2 + k_2^2 + k_3^2)}\Bigg[&\left(\frac{\frac{1}{C - E_1} - A_1}{B_1}\right)^4 (1 - (k_2^2 + k_3^2)) + \\
&+ \left(\frac{\frac{1}{C - E_2} - A_1}{B_1}\right)^4 (1 - (k_3^2 + k_1^2)) + \left(\frac{\frac{1}{C - E_3} - A_1}{B_1}\right)^4 (1 - (k_1^2 + k_2^2))\Bigg]\Bigg\}^{1/2}.
\end{aligned} \tag{8}$$

When $l/d \approx 500$, as in our case, then $k_1 \approx k_2 \approx k_3 \approx 0$ as has been demonstrated [2, 3], so that expressions (7) and (8) become

$$v_x = \left\{\frac{1}{2}\left[\left(\frac{\frac{1}{C - E_2} - A_1}{B_1}\right)^4 - \left(\frac{\frac{1}{C - E_3} - A_1}{B_1}\right)^4 - \left(\frac{\frac{1}{C - E_1} - A_1}{B_1}\right)^4\right]\right\}^{1/2},$$

$$v_y = \left\{ \frac{1}{2} \left[\left(\frac{\frac{1}{C-E_1} - A_1}{B_1} \right)^4 - \left(\frac{\frac{1}{C-E_3} - A_1}{B_1} \right)^4 - \left(\frac{\frac{1}{C-E_2} - A_1}{B_1} \right)^4 \right] \right\}^{1/2},$$

$$v_z = \left\{ \frac{1}{2} \left[\left(\frac{\frac{1}{C-E_2} - A_1}{B_2} \right)^4 - \left(\frac{\frac{1}{C-E_1} - A_1}{B_1} \right)^4 - \left(\frac{\frac{1}{C-E_3} - A_1}{B_1} \right)^4 \right] \right\}^{1/2}, \quad (9)$$

$$v = \left\{ \frac{1}{2} \left[\left(\frac{\frac{1}{C-E_1} - A_1}{B_1} \right)^4 + \left(\frac{\frac{1}{C-E_2} - A_1}{B_1} \right)^4 + \left(\frac{\frac{1}{C-E_3} - A_1}{B_1} \right)^4 \right] \right\}^{1/2}.$$

These expressions for determining the mean-velocity vector and its components include calibration constants A_1, B_1, and C. Despite the multitude of factors influencing the heat transfer at the transducer wires, however, it is possible, through precise control of the manufacturing process, to produce a probe with almost identical wires and thus with almost identical calibration constants A_1, B_1, and C respectively. This shortens appreciably the computation time required by the algorithm (9) and also the calibration time. For calibration, the probe wires are successively placed in the stream perpendicularly to the direction of flow and the thermoanemometer output voltage is plotted as a function of the stream velocity for each wire at the operating overheat temperature. The constants A_1 and B_1 are evaluated by the least-squares method [4] on an "Élektronika DZ-28" computer. Programs have also been written for implementing this method on a BZ-21 calculator. For the probe used in this study these constants were $A_1 = 0.193\ V^{-1}$ and $B_1 = 0.0162\ sec \cdot V^{-1}$. The constant C was determined from relation (5a), with $\alpha = 2.7$ for the instrument built here. The electrical resistance R_W of each wire in the overheat state was determined from the condition of balance in the corresponding Wheatstone bridge. In this case the probe was replaced by an MSR-63 resistor bank for balancing the instrument bridge. The readings of the MSR-63 resistor bank at balance were equal to the corresponding probe resistance R_W in the overheat state. The value of the constant C for the given instrument with probe was 11.03 V.

For the purpose of checking the performance of the algorithm and determining the angular range of the probes for the stipulated accuracy of measurement of the mean-velocity vector in a stream, a series of experiments was performed in which the modulus of the mean-velocity vector and its components were measured as functions of angle of attack at stream velocities of 10 and 20 m/sec, respectively. These experiments were performed in an aerodynamic test stand, with the probe mounted on a rotatable coordinate plotter in the main gap for recording the displacments in two mutually orthogonal planes.

The relatively long holder, ≈200 mm, and the complexity of the precision mounting for the probe made a precise alignment of the probe axis according to the direction of the stream impossible.

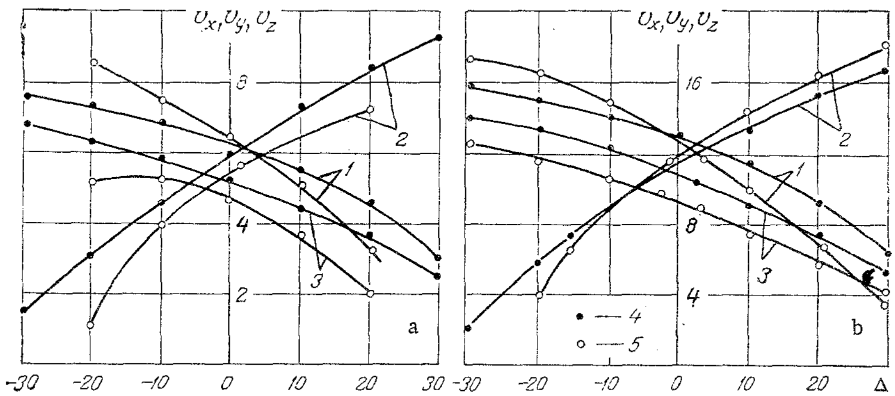

Fig. 3. Experimental and theoretical dependence of components of velocity vector on angle of attack Δ (deg), for (a) v = 10 m/sec and (b) v = 20 m/sec; 1) v_x, 2) v_y, 3) v_z, 4) calculation, 5) experiment; $\alpha = 40°$.

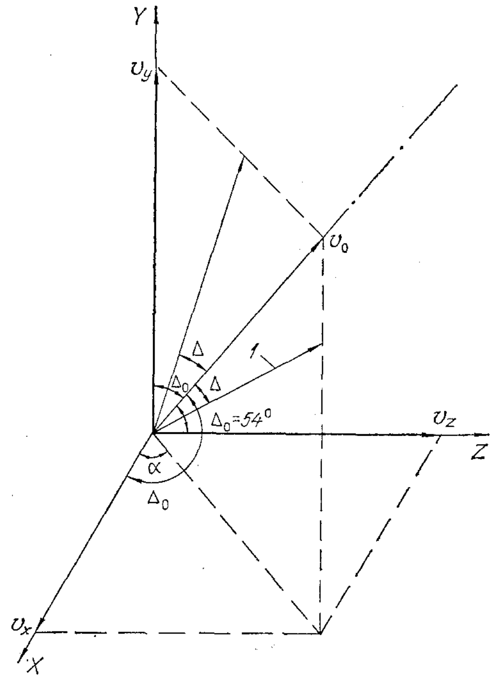

Fig. 4. Disposition of probe relative to axes of coordinates in experiment: 1) holder axis, $v_x = v \sin(\Delta_0 \pm \Delta) \cos \alpha$, $v_y = v \cos(\Delta_0 \pm \Delta)$, $v_z = v \sin(\Delta_0 \pm \Delta) \sin \alpha$.

The error due to imprecise mounting was $\approx 5°$, within the required 10% accuracy of v_x, v_y, v_z, and v determination, so that no attempts were made to find a way to mount the probe more precisely. If it should be necessary, however, then, of course, this problem must be carefully considered. The graphs in Fig. 3 show how the components of vector v change as the vector deviates from its initial orientation along the holder axis. The vector was made to deviate in the plane passing through one of the wires, say wire 2 (OY axis), and the holder axis up to an angle $\Delta = \pm 30°$ from the holder axis, with the latter held at an angle $\Delta_0 = 54°$ to all wires. The theoretical dependences of the coordinates characterizing the components of the velocity vector on the angle of attack Δ was calculated according to the relations shown in Fig. 4. The graphs in Fig. 3 indicate a close agreement (within the stipulated accuracy) between the theoretical relations and the experimental curves plotted according to the given algorithm.

As the velocity of the stream decreases, moreover, the error of its determination increases. This is so because of the increasing error in the determination of the calibration constants A_1 and B_1 by the method of least squares at low velocities of the stream. A more accurate determination of these two constants in the King equation requires, evidently, that readings for calibration at low velocities be taken in steps not larger than 1 m/sec.

NOTATION

v, velocity of an air stream; v_{eff}, effective cooling velocity of an air stream; v_x, v_y, v_z, components of the velocity vector; θ and Δ, angle between velocity vector v and a probe wire; ψ, angle between vector v and a plane normal to the wire axis; R_a, electrical resistance of a wire at ambient temperature; R_w, electrical resistance of an overheated wire; A, A_1, B, B_1, C, calibration constants; E, thermoanemometer output voltage; a, thermoanemometer proportionality factor; k_1, k_2, k_3, empirical constants; and α, angle between wire 1 and the plane passing through wire 2 and the axis of the probe holder.

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NONISOTHERMAL FLOW OF CHEMICALLY REACTING MEDIA WITH VISCOSITY DEPENDING ON TEMPERATURE AND PRESSURE

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The article investigates heat exchange and resistance when non-Newtonian, chemically reacting liquids, whose viscosity depends on the flow rate, the temperature, the pressure, and the degree of conversion, flow through flat pipes.

Flow of highly viscous materials (e.g., molten thermosets or rubber mixtures) occurs at high temperatures and pressures, and it is accompanied by considerable dissipative heat liberation, and also by chemical reactions (e.g., the reactions of curing or vulcanization). As a result of these reactions, the viscosity of the materials increases, and this imposes certain limits on the duration of their state of viscous flow. This is of great technological importance because it makes it possible to establish how long the material can be permitted to remain in the working components of the processing equipment.

The authors of [1] and [2] examined questions of nonisothermal flow of chemically reactive media; their solutions are correct only for the flow of Newtonian liquids, and they do not take the effect of the pressure on the physical properties into account.

Below we examine nonisothermal flow of non-Newtonian liquids in a flat pipe on whose outer surface heat exchange with the environment proceeds according to Newton's law (the heat transfer coefficient and the ambient temperature are known). In consequence of the high viscosity of the liquid and its low thermal diffusivity (which is a characteristic feature of many converted polymer materials), the generalized Reynolds criterion does not exceed 10^{-2} whereas the generalized Prandtl number attains 10^5. Therefore, the hydrodynamic initial section is practically lacking, and the speed profile at the pipe inlet may be taken to be fully developed, with no slip on the wall. The temperature distribution at the inlet is taken as uniform over the entire cross section of the channel. As the physical model of heat exchange of the liquid with the inner pipe walls we adopted the model of the thermal boundary layer [3, 4] which assumes that with high Graetz numbers, heat exchange proceeds only in the region near the wall, and that this region increases with increasing distance from the pipe inlet, until the liquid is heated completely over the entire cross section of the channel. It is also assumed that in the material flowing through the pipe, there occurs a chemical reaction of first order. The flow pattern of the liuqid in accordance with the adopted assumptions is shown in Fig. 1.

Assuming that between the stress extra tensor and the strain rate tensor there exists a correlation in the form of Ostwald de Vila's exponential equation, we represent the system of differential equations describing the process of nonisothermal flow of a rheologically complex liquid in the Cartesian system of coordinates in the following manner:

$$\rho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y}\right) = -\frac{dP}{dx} + 2\frac{\partial}{\partial x}\left[K\left|\frac{\partial v_x}{\partial y}\right|^{n-1}\frac{\partial v_x}{\partial x}\right] + \frac{\partial}{\partial y}\left[K\left|\frac{\partial v_x}{\partial y}\right|^{n}\operatorname{sign}\left(\frac{\partial v_x}{\partial y}\right) + K\left|\frac{\partial v_x}{\partial y}\right|^{n-1}\frac{\partial v_y}{\partial x}\right], \quad (1)$$

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